GLOBAL SCALING LAWS IN 2D OPEN BILLIARDS

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We investigate numerically 2D open billiard systems in rectangular, triangular, and honeycomb lattices with emphasis on scaling laws of static statistical quantities such as a free path length and its moment. About twenty years ago, Sinai proved that the open billiard systems exhibit ergodic behavior provided that a scatterer has an everywhere dispersing shape. This ergodicity in principle stands for the fact that a Liouville measure can be realized as a natural measure as in classical ergodic Hamiltonian dynamical systems. Note that the billiard system belongs to a Hamiltonian dynamical system with a box-type potential. This Liouville measure simply represents a volume element in phase space in the billiard problem. Therefore in ergodic open billiard systems several simple statistical laws hold. For example, in T^2 torus, a pressure distribution as a result of an infinite number of impacts of a particle satisfies an ideal gas law, when we consider a rectangular lattice, a triangular lattice, and a honeycomb lattice. More important is the fact that a statistical average of a free path length can be expressed by

$$< l >= \frac{\pi S}{\gamma_0}, \tag{1}$$

where S and γ_0 represent an allowable area of motion of a particle within a unit cell, and a total perimeter length of an everywhere dispersing scatterer whose form may be arbitrary, respectively.

Equation (1) plays a significant role in investigating numerically various static statistical quantities such as time averages of n-th power of a free path length. This is because Eq.(1) is valid for a rectangular lattice, a triangular lattice, and a honeycomb lattice, as we have confirmed numerically in the case of an ellipsis scatterer. This fact leads us to the investigation of various time averaged quantities under the assumption that the variable $\frac{\pi S}{\gamma_0}$ would be regarded as a scaling variable.

For the purpose of confirming this anticipation, we have investigated numerically the behavior of $< l^n >, \quad (n = 2, 3)$. The log-log plot of $< l^n >$ versus $< l >$ reveals that, although the directly obtained curve shows abrupt bursts from place to place, the enveloping curve drawn from below the obtained curve has a constant slope irrespective of the arbitrary change of the scatterer, but depends on the type of the lattices under consideration at least numerically. Therefore we may call the slope a scaling exponent. Of course, the slope of this enveloping curve changes in a slightly irregular way, because the curve is obtained only numerically. It is, however, noteworthy to point out that this enveloping curve remains almost the same for 4 to 8 changes of the form of the ellipsis scatterer in any type of lattices. The results are in the following.

Scaling Exponents			
	rectangular lattice	triangular lattice	honeycomb lattice
$< l >$	1.00	1.00	1.00
$< l^2 >$	2.16	2.18	2.22
$< l^3 >$	3.43	3.52	3.61

However, the lattice dependence of the slope is a little ambiguous, and the slope in each case takes almost the same value. Therefore, in order to assert that the lattice dependence of the slope actually exists, we expect that we would probably have to carry out numerical simulations more carefully in each type of lattice, or we would have to resort to theoretical analysis.

We finally discuss the results extended to 3D open billiard systems [1] briefly.

[1] S. Koga, Prog. Theor. Phys. **93** (1995),19